Fermi National Accelerator Laboratory

FERMILAB-Pub-82/26-THY
March, 1982

Do Light Colored Scalars Exist?

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ABSTRACT

We examine the general question of the properties of light, neutral colored spin-zero particles in QCD. Models with spontaneous breaking of QCD at very large distances, such as that of DeRújula, Giles, and Jaffe and the SO(3) scheme of Slansky, Goldman and Shaw require such light colored Higgs scalars. These scalars will form color singlet hadronic bound states at short distances and we estimate bound state masses, decay widths, and production rates in processes such as $\psi\to\gamma+X$ within the MIT bag model. The resulting states are expected in the mass neighborhood $\sim$1.5 GeV and should resemble glueballs.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

I. INTRODUCTION

It is generally accepted that QCD is the correct theory of quark and gluon interactions, and that quarks and gluons are confined into color singlet hadronic bound states. Taken together with leptons and the known vector bosons it is quite remarkable that up to a mass scale of the order of 20 GeV there is no direct experimental evidence for the existence of elementary, point-like spin-zero objects. An electrically neutral, light, colored spin-zero particle would be bound into a hadronic composite state and would be quite difficult to observe.

Some motivation for considering such objects can be gleaned from theoretical interpretations of the possibility of the existence of fractionally charged particles, as reported in recent experiments [1]. Since all color singlets have zero triality, the conventional charge and color assignments of quarks, leptons, and vector bosons guarantees the absence of fractionally charged particles in the presence of confinement. If one is led, however, to accept the reports of the observation of fractionally charged objects, then one must consider either a departure from the conventional relationship between triality and charge assignments, e.g., either there may exist fractionally charged leptons or non-fractionally charged quarks, or that color confinement is not an exact physical phenomenon but is violated, albeit sufficiently weakly to have escaped direct experimental detection.

There exist two models in the literature adopting the latter point of view. DeRújula, Giles and Jaffe [2] (DGJ) propose that SU(3) of color is broken by three color triplets of Higgs scalars in an

$SU(3)_{global}\times SU(3)_{color}$ potential down to a residual global SU(3) of color. Since here the only exact symmetry of the world emerges as a global SU(3), the quarks become liberated at large distances and can account for the observation of fractional electric charge. However, since this model requires three electrically neutral color-triplet Higgs scalars it already abandons the conventional triality-electric charge relationship, as well as color confinement.

Slansky, Goldman and Shaw, (SGS) [3], have suggested a model in which $SU(3)_{color}$ breaks to a local SO(3). This can be accomplished with an SU(3) 27-dimensional neutral Higgs multiplet, which has zero triality, and thus the triality-electric charge relationship is preserved. (A more economical method would be to use the color sextet which violates triality, ab initio). In this model, the quarks become vector triplets of SO(3). Since SO(3) is assumed to be confining, only SO(3) singlets can ultimately exist and hence diquarks are allowed as large distance bound states and could account for the observation of fractional charge.

In general, models of spontaneously broken QCD produce fractionally charged objects at large distances, $r \gg 1/M$, where M is the mass of the gluons whose color charges do not annihilate the vacuum. Since there exist extremely tight experimental limits on the production of fractionally charged states it is necessary to have a dynamical picture in which production of such exotics is suppressed in conventional accelerator experiments. Suppose that QCD is broken to a local group G at an energy scale M. At a distance scale on the order of 1/M there will exist states which are singlets under G but have nontrivial $SU(3)_{color}$ transformation properties. The conventional picture is a string model in which the string must be stretched to the distance 1/M without breaking in order to form these states. The probability that a QCD string can be stretched a distance 1/M without disintegrating into ordinary pions, etc., is described by a vacuum persistence probability [4]:

$$P_r \sim \exp(-2\times10^4(\mathrm{MeV})^2/M^2) \quad . \tag{1.1}$$

In order to suppress the production of fractionally charged states to the order required by recent PEP limits [5], the parameter M must be adjusted to an extremely small value. Experimentally, at an energy Q = 29 GeV:

$$R_{q_{2/3}} = \frac{\sigma(e^+e^-\to\bar{q}_{2/3}q_{2/3})}{\sigma(e^+e^-\to\mu^+\mu^-)} \lesssim .01 \tag{1.2}$$

for charge 2/3 objects and for a wide range of quark masses. We approximate:

$$R_{q_{2/3}} \sim \sum_{\substack{\text{colors}\\ \text{flavors}}} (2/3)^2 P_r \tag{1.3}$$

and using eq.(1.1), we find $M \lesssim 60$ MeV. Hence, the scale at which $SU(3)_{\mathrm{color}}$ can break down to something else is extremely small.

Of course, such schemes are speculative and may contain other theoretical problems as well. Virtually any of the properties of the large distance fractionally charged states will be impossible to estimate reliably without a detailed knowledge of the breaking dynamics due to the high sensitivity upon the parameter M through the exponential of eq.(1.1). As witness to this fact, recently Kolb, Steigman and Turner [6] have attempted to estimate the relic abundance of fractionally charged diquarks in the SGS model. Their result for the number of diquarks per nucleon, ρ, is precisely $10^{-50}<\rho<10^{10}$!

In the present paper we will examine the properties of confined light scalars in QCD. These objects would constitute an indirect test of the broken $SU(3)_{color}$ schemes, all of which require the existence of light colored Higgs scalars. Furthermore, the properties and systematics of these new hadrons are not subject to the enormous uncertainties inherent in the large distance fractionally charged states. To estimate the properties of these objects, we may turn to any standard hadronic bound-state model, e.g., the MIT bag model [7]. Our intuitive expectations are roughly confirmed for these bound states as they turn out to be in the mass range of about 1 to 2 GeV with decay widths ranging from a few MeV to much greater than 100 MeV, depending strongly upon the color representations of their constituents. These bound states are prime candidates for the process $\psi\to\gamma+X$ [8]. However, there are various uncertainties in the bag model estimates themselves, particularly for the higher color representations of the constituent scalars and it is conceivable that some of the objects are too heavy to appear in ψ decay.

An important subtlety we shall encounter is the possibility that the bag state containing two high dimensional color representations is itself tachyonic and thus leads to a condensation in the vacuum. We shall argue that such condensates are acceptable if there is no continuous symmetry associated with the bag state, in which case the physical excitation remaining has a positive, non-zero mass squared, (i.e., no Goldstone bosons appear).

Clearly there are many possible choices of G in the breaking scheme SU(3)→G beyond those of DGJ and SGS. G can be any one of the following: SO(3), SU(2)×U(1), SU(2), U(1)×U(1), U(1), $SU(3)_{global}$, any other global symmetry, or nothing at all. In general, these will require different Higgs structures to implement. We restrict ourselves only to those representations that are consistent with asymptotic freedom. Hence, we allow scalars transforming as the {3}, {6}, {8}, {10}, {15}, {21}, and {27} dimensional representations of SU(3). In Section II, we calculate the masses of the bound states of these scalar fields and in Section III we examine the question of breaking QCD with vacuum expectation values of scalar fields. We look at this problem in the bag model and suggest a criterion for when the spontaneous breakdown of QCD is allowed. In Section IV, we calculate hadronic decay widths and production cross sections and we derive the excitation spectrum of the bound states. Finally, in Section V, we examine the experimental evidence restricting the existence of such scalar-scalar bound states.

II. SCALAR-SCALAR BOUND STATES

The mass of the 0^{++} bound state can be calculated using the MIT bag model with two scalar fields confined to a static spherical cavity of radius R_0. Let ϕ_r denote a scalar in the r-dimensional color representation, (which may be either real or complex), and Φ_r a color singlet bound state containing two of the ϕ_r. In the absence of perturbative gluonic corrections, a complex scalar field in the bag has the Hamiltonian:

$$H = \int d^3\vec{x}\{|\frac{\partial\phi_r}{\partial t}|^2 + \vec{\nabla}\phi_r^+\cdot\vec{\nabla}\phi_r + \mu^2|\vec{\phi}_r|^2 + B\} \quad , \tag{2.1}$$

where μ^2 may be either positive or negative and the integral extends over the bag volume and B is the bag constant. The resulting equations of motion and boundary conditions are:

$$\text{(i)}\quad (\Box+\mu^2)\phi_r(\vec{r},t) = 0 \quad , \qquad r<R_0 \tag{2.2a}$$

$$\text{(ii)}\quad \phi_r(\vec{R}_0,t) = 0 \quad , \text{ and} \tag{2.2b}$$

$$\text{(iii)}\quad \partial_\mu\phi_r(\vec{R}_0,t)^+\partial^\mu\phi_r(\vec{R}_0,t) = -B \quad . \tag{2.2c}$$

A discussion of the derivation of these constraints is given in ref. [7]. Condition (iii) follows upon demanding zero net momentum flow through the static bag wall and fixes the solution's normalization for arbitrary R_0 in terms of B and R_0.

Taken together, eqs.(2.2a,b,c) provide a solution for ϕ_r for any value of the bag radius R_0, but do not yield a quantization of the energy spectrum. For a complex field ϕ_r with angular quantum numbers ℓ and m, we have the general solution:

$$\phi_r(\vec{r},t) = \eta(B,R_0)j_\ell(K_\ell r)Y_{\ell m}(\theta,\phi)e^{i\omega_K t} \quad , \tag{2.3}$$

where $\omega_K^2 = K_\ell^2 + \mu^2$, $\eta(B,R_0)$ is a determined normalization factor, and

$j_\ell(K_\ell R_0) = 0$. For the case $\ell=0$,

$$\phi_r(\vec{r},t) = \frac{\sqrt{BR_0^4}}{\pi p}\,\frac{\sin Kr}{r}\,e^{i\omega_K t} \quad , \tag{2.4}$$

where $K = \pi p/R_0$, p is an integer, and the energy is,

$$E_{TOTAL} = \frac{16}{3}\,\pi R_0^3 B \quad , \tag{2.5}$$

which is independent of the particular state in the bag. Hence, R_0 is not fixed in our solution and the state of minimum energy is the state where R_0 goes to zero.

The proper quantization of the small oscillations about a given classical solution to the bag can be carried out as in ref. [7]. This again fixes the normalization of the solution and defines the quantized values of R_0 and hence E. Unfortunately, there is an overall zero-point energy which remains unspecified. We refer the reader to the literature for this discussion [7] and we opt instead for a "quick-fix" which leads to a reasonable physical picture and supplies the extra needed constraints. We fix R_0 by demanding that the constituent fields have a particle number normalized to unity:

$$Q = -i\int d^3\vec{r}\,\phi^+\vec{\partial}_0\phi = \frac{4BR_0^4}{p} \equiv 1 \quad , \tag{2.6}$$

where we have explicitly evaluated Q for the solution in eq.(2.4). Adopting the normalization requirement of eq.(2.6) gives the result for the energy:

$$E = \frac{4}{3}\pi R_0^3 B + \sum_{i=1}^{m} \sqrt{K_{\ell_i} + \mu^2} \quad ; \quad j_{\ell_i}(K_{\ell_i} R_0) = 0 \quad , \tag{2.7}$$

for m constituents each in the indicated mode. The minimum of E with respect to R_0 now determines the mass of the bound state and for $\mu^2=0$ the result for two particles each in the $\ell=0$ lowest radial mode is:

$$E = \frac{8}{3} 2^{1/4}\pi B^{1/4} = 9.96B^{1/4} \quad . \tag{2.8}$$

B is determined by analyses that fit the bag model spectrum to the hadronic spectrum and is numerically, $B^{1/4} = .145$ GEV [9]. Hence, in the naive approximations leading to eq.(2.8), we expect the 0^{++} state to weigh in around 1.44 GeV.

The result of eq.(2.8) for the 0^{++} bound state assumes $\mu^2=0$. In Fig. 1 we plot the mass of the bag containing two ϕ_r, determined by minimizing eq.(2.7) numerically, against $|\mu|$ in units of $B^{1/4}$. Branch I is for constituents with $\mu^2>0$ and is seen to rise with increasing μ, becoming asymptotic to the free particle masses 2μ. Of course for $|\mu|/B^{1/4}\geq 10$ we are no longer justified in using the bag model alone and should resort to a potential model. Branch II of Fig. 1 shows the bag mass for constituents having $\mu^2<0$. We will elaborate upon this in Section III, but we note presently that $|\mu|/B^{1/4}\geq 2.7$ corresponds to the bag mass acquiring an imaginary part and we believe that this is a signal that the Φ state is becoming tachyonic, i.e., that it will condense by acquiring a vacuum expectation value. This corresponds to a Φ with a mass near 1 GeV. For still larger values of $|\mu|$, we believe there will persist a Φ state residual excitation with roughly this mass.

In Section III, however, we will discuss the related question of how the actual breaking of $SU(3)_{color}$ is realized by the $-\mu^2$ of the constituents in the bag model.

When the Φ state becomes tachyonic, either by virtue of a negative mass-squared of the constituents or by gluon exchange, it is presumably described by an effective potential whose first few terms take the form:

$$V(\Phi) = \frac{-m^2}{2}\Phi^2 + \frac{\lambda}{4}\Phi^4 + \ldots \tag{2.9}$$

Of course, Φ will develop a vacuum expectation value, $\Phi \rightarrow v + \hat{\Phi}$, but the only broken symmetry in (2.9) is a discrete one (at most) and thus the leftover excitation, $\hat{\Phi}$, will not be a Goldstone boson but will have a mass near m. Hence condensation of the 0^+ state in general does not remove the 0^+ state from the spectrum and we still expect such an object in the range of 1 to a few GeV.

We have set the constituent mass of the scalars ϕ_r to zero. This approximation is easily justified by the following simple model. Suppose that the breakdown of QCD is caused by the vacuum expectation value, (VEV), of a single scalar field, ϕ. Then the potential energy may be written as

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4, \tag{2.10}$$

and the minimum of the potential occurs when

$$\langle\phi\rangle = \sqrt{\mu^2/\lambda} \quad . \tag{2.11}$$

The VEV of ϕ causes the gluon to obtain a mass m_g,

$$m_g \sim g \langle\phi\rangle \sim g\mu/\sqrt{\lambda} \quad . \tag{2.12}$$

Thus,

$$\mu \sim \frac{m_g\sqrt{\lambda}}{g} < \frac{4\pi m_g}{g} , \tag{2.13}$$

since unitarity arguments require that $\lambda^2/16\pi^2 \leq 1$. For small gluon masses, (say $m_g \sim 50$-100 MeV), the strong coupling constant g is presumably very large since we are in the non-perturbative confinement region and so μ is very small. The same argument will hold in the more general case where the scalar field transforms non-trivially under the SU(3) gauge group.

There exist further corrections to the expression for the bag mass of Eq. (2.8). These include (a) the presence of quantum fluctuations of all colored fields in the cavity, (b) gluon exchange and self energy corrections, (c) center of mass motion effects, (d) a possible dependence of B upon the constituents' color representation r and (e) mixing of the Φ_r with the 0^{++} glueball. These we briefly discuss here.

(a) The bag will contain the zero point fluctuations of all colored fields in the theory including quarks, gluons and scalars. This leads to a "Casimir effect" correction to the energy. For quarks and gluons this correction takes the form [9]:

$$E_{q,g} = \alpha\Lambda^4 R_0^3 + Z_0/R_0 , \tag{2.14}$$

where Λ is a divergent cut-off and R_0 is the bag radius. For scalar fields one finds:

$$E_{\text{scalars}} = \alpha\Lambda^4 R_0^3 + \beta\Lambda^3 R_0^2 + Z_0/R_0. \tag{2.15}$$

Conventionally one treats the α-terms above as corrections to the bag constant and the Z_0/R_0 term is included phenomenologically. Hence, one does not have to deal with the divergent cut-off in this prescription. For scalars, however, the β term appears which cannot be renormalized away and one is faced with having to interpret the cut-off Λ. This requires some care.

For very short-wavelength quanta we expect a natural cut-off from asymptotic freedom and thus Λ is expected to be finite. For intermediate wavelengths we must consider the finite bag wall thickness which presumably becomes transparent for sufficiently short wavelengths. An analogy is afforded by the Casimir force between two parallel conductors such as copper. This too leads to a divergence as in Eq. (2.14). However, this divergence is due to the assumption of a perfect conductor which does not apply in reality for all wavelengths. Certainly photons with wavelengths less than a copper atomic size scale are unaware of the conducting boundary conditions, and the physical cut-off is of the order of the plasma frequency of the medium. In QCD we expect that the cut-offs in Eq. (2.14) are of order Λ_{QCD} in reality and thus $\Lambda \sim \Lambda_{QCD} \sim 100$ MeV. $\Lambda^3\beta$ can in principle be determined by including this term in fits to the hadronic spectrum.

The term Z_0/R_0 is included in bag model hadronic spectrum fits with the result that $Z_0 \approx -2.0$ [9]. Since a typical bag radius is $R_0 \simeq 5\ \text{GeV}^{-1}$ we see that the identification of $\Lambda \sim \Lambda_{QCD}$ allows the neglect of the β term provided:

$$|Z_0/R_0| \backsim 0.4 \text{ GeV} >> |\beta\Lambda^3_{QCD}R^2_0| \backsim |\beta(0.025)|\text{GeV} , \tag{2.16}$$

and hence we require $|\beta|<<16$. β is proportional to the dimensionality of the color representation r and for very large r we expect this term to become important. It would be of interest to apply a large β-term to the calculation of the normal hadronic spectrum and to attempt an estimate of β. This could further bound the light scalar content of QCD. We however neglect the β term completely in the mass estimates of bound states given below.

When we include the zero point energy term Z_0/R_0, the equation for the energy of n scalars in a bag, Eq. (2.7), becomes

$$E = \frac{4}{3}\pi R_0^3 B + \sum_{i=1}^{n}\sqrt{K_i^2 + \mu^2} + Z_0/R_0. \tag{2.17}$$

As before, we minimize the energy with respect to R_0 to obtain, (for $\mu^2=0$),

$$E = \frac{4}{3}(4\pi B)^{1/4} \sum_{i=1}^{n}(n_i + Z_0)^{3/4} , \tag{2.18}$$

where $K_i = n_i/R_0$. With $Z_0=-2$, the mass of the 0^{++} bound state is reduced to 1.08 GeV by this correction.

(b) Gluon exchange is expected to be a substantial correction to the bound state mass, especially for constituents in the larger color representations since it generally scales with the Casimir operator $C_2(r) = \sum_{aj}(\Lambda^a_{ij}/2)(\Lambda^a_{ji}/2)$. One must include the effects of gluon exchange between two constituents as well as the corrections to the constituents' self-energy due to the finite size of the bag, (see Fig. 2). One generally considers these effects together as a color-generalization of the electromagnetic self-energy of a charge distribution:

$$\Delta E = \frac{1}{2} \iint \frac{\rho(x)\rho(y)d^3xd^3y}{|\vec{x}-\vec{y}|} , \tag{2.19}$$

where $\rho^a(x) = \rho_1^a(x) + \rho_2^a(x)$ is the total local color charge density from the two constituents. This expression is valid in the static approximation in which the constituents always remain in the same wave-function for all time t.

By elementary manipulations, this contribution to the energy can be rewritten in terms of the electrostatic self-energy of the electric field:

$$\Delta E = \frac{1}{8\pi} \int d^3x \, \{\vec{E}^a(x)\cdot\vec{E}^a(x)\} . \tag{2.20}$$

For the 0^{++} state both constituents have equivalent wave functions and we may write for the gluon exchange energy:

$$\Delta E(0^{++}) =$$

$$\frac{1}{2} \iint \langle 0^{++}|(\rho_1^A(x)+\rho_2^A(x))(\rho_1^A(y)+\rho_2^A(y))|0^{++}\rangle \frac{1}{|\vec{x}-\vec{y}|} d^3xd^3y \tag{2.21a}$$

$$= \iint \left(\langle 0^{++}|\rho_1^A(x)\rho_1^A(y)|0^{++}\rangle + \langle 0^{++}|\rho_1^A(x)\rho_2^A(y)|0^{++}\rangle\right) \frac{d^3xd^3y}{|\vec{x}-\vec{y}|} , \tag{2.21b}$$

where $\rho_{1(2)}^A(x)=i\phi_{1(2)}^+\partial_0 \, \lambda^a/2 \, \phi_{1(2)}$ for distinguishable constituents. For a constitutent wave function of the form

$$\psi(r,t) = \eta\, j_0(Kr)e^{i\omega_K t}\;, \tag{2.22}$$

we obtain

$$\iint \langle 0^{++}|\rho_1^A(x)\rho_1^A(y)|0^{++}\rangle \frac{d^3x d^2y}{|\vec{x}-\vec{y}|}$$

$$\simeq \iint \langle 0^{++}|\rho_1^A(x)|0^{++}\rangle\langle 0^{++}|\rho_1^A(y)|0^{++}\rangle \frac{d^3x d^3y}{|\vec{x}-\vec{y}|} \tag{2.23a}$$

$$= \eta^2 4\omega_K^2 \iint d^3x d^3y\, \frac{j_0(Kx)^2\; j_0(Ky)^2}{|\vec{x}-\vec{y}|}\;\; \delta^{if}\, \frac{\lambda^A}{2}_{ij}\, \frac{\lambda^A}{2}_{jl}$$

$$\cdot\, \frac{\delta^{\ell\hat{i}}}{\dim(r)}\;, \tag{2.23b}$$

where use is made of the approximation of inserting only the 0^{++} intermediate state (static approximation).

Similarly we obtain:

$$\iint \langle 0^{++}|\rho_1^A(x)\rho_2^A(y)|0^{++}\rangle \frac{d^3x d^3y}{|\vec{x}-\vec{y}|} =$$

$$4\eta^2\omega_K^2 \iint \; j_0(Kx)^{\;2}\; j_0(Ky)^{\;2}\; \frac{d^3x d^3y}{|\vec{x}-\vec{y}|}\;\; \frac{\lambda^A}{2}_{ij}\; \frac{(\tilde{\lambda}^A_{ml})}{2}\; \frac{\delta^{im}\delta^{jl}}{\dim(r)} \cdot \varepsilon\;.$$

For scalars in a real representation of SU(3) of color and a 0^{++} color singlet, we find that $\tilde{\lambda}^A_{ml}=\lambda^A_{ml}$ and $\varepsilon=1$, but that $\lambda^{AT}=-\lambda^A$ and thus these two terms cancel by virtue of their group theoretical coefficients. For a complex representation for the constituents, we find that $\tilde{\lambda}^A_{kl}= \lambda^{A*}_{kl}$ and $\varepsilon=-1$ so again the terms cancel. For any state that has positive charge conjugation involving complex constituents, or has even parity involving real constituents in a color singlet the same cancellation will always occur. In the latter case this is just the result of Bose statistics which forces the $\rho_1\rho_1$ term in the self energy correction to have the same magnitude as the $\rho_1\rho_2$ term.

This is also a consequence of our neglect of intermediate states that are not in the same configuration as the ground state. Since the net contribution of gluonic energies to the bag is expected to be large compared to the level spacing we do not really trust the static approximation. For example, keeping only the effect of Fig. (2a), the one-gluon interparticle exchange, our net gluonic correction to the energy of Eq. (2.8) would be given by:

$$\begin{aligned}\Delta E &= -g_s^2 C_2(r) \frac{BR_0^4}{\pi^4} \int d^3x d^3y \frac{\sin K|x|}{K|x|} \frac{\sin K|y|}{K|y|} \frac{1}{|\vec{x}-\vec{y}|} \\ &= -\frac{4C_2(r)}{R_0} \alpha_s \left(\int_0^{2\pi} \frac{\sin x}{x} dx\right)^2 \\ &\simeq -(5.68\ C_2(r)) \frac{\alpha_s}{R_0} ,\end{aligned} \tag{2.24}$$

(where g_s is the strong coupling constant). If we parameterize this correction by a factor Z such that $-1 \leq Z \leq 1$ and

$$\Delta E = -\frac{Z\alpha_s}{R_0} (5.68\ C_2(r)) , \tag{2.25}$$

the mass of the 0^{++} ground state then ranges from a tachyonic unphysical value for $Z \sim 1$ to a very large mass on the order of 2 GeV for $Z=-1$ and $r=3$ (for $Z=-1$ and $r=27$, this mass becomes 4.5 GeV).

Further contributions would be expected from the repulsive $\lambda\phi^4$ interaction of Fig. (3a) which is on a footing with one loop QCD self energy corrections. These we do not discuss but we expect them to be potentially important.

(c) Center of mass motion will be discussed in Section IV.

(d) The bag constant may depend upon the color representation of the constituents. Since bag masses scale as $B^{1/4}$ this dependence is weak.

We assume as a bound that B is scaled by $C_2(r)$ and note that this increases the mass of the bound state of two {27}'s, Φ_{27}, from 1.08 GeV to 1.20 GeV and so this is not a significant effect.

(e) In addition to the one-gluon exchange diagram of Fig. 2, bound states of two identical scalar particles can also receive masses from the annihilation graphs of Fig. 3b. Indeed, it has been suggested that it is similar annihilation graphs which cause the η' to be much more massive than the value calculated from the naive bag model [10]. Since the Φ_r have masses near that of the 0^{++} glueball, (in the naive bag model, the 0^{++} glueball has a mass of 980 MeV) [11], the mixing between the Φ_r and the glueball, Fig. 3b, may be significant. We estimate this mixing by comparing it with the mixing of the η' with the 0^- glueball. (Since the η' is a flavor singlet, it mixes with the 0^- glueball via a diagram analogous to Fig. 3b). The overlap of the 0^- glueball $(G\tilde{G})$ with the η' has recently been calculated in the bag model by Carlson and Hansson [10] to be,

$$\langle G\tilde{G}|\eta'\rangle \simeq 0.08 \ . \tag{2.26}$$

Since this effect is proportional to T(r) for the color triplet quarks in the η', we estimate,

$$\langle GG|\Phi_r\rangle \simeq \frac{1}{4}\ \frac{T(r)}{T(3)}\ \sqrt{\frac{\dim(3)}{\dim(r)}}\ (0.08) \quad , \tag{2.27}$$

(the one quarter is the familiar kinematic result of replacing fermions by scalars and the factor of dim(r)/dim(3) is the result of the differing wavefunction normalizations of the Φ_r and the η'). For the bound state of two color {27}'s we estimate,

$$\langle GG|\Phi_{27}\rangle \simeq 0.36, \qquad (2.28)$$

a potentially large effect. (For the color triplets, and the smaller dimensional SU(3) representations, this effect is much smaller, eg. $\langle GG|\Phi_3\rangle \simeq 0.02$ and $\langle GG|\Phi_{10}\rangle \simeq 0.17$.)

However, this naive scaling of the results for the η' may be incorrect and in any case, the contribution to the masses of the Φ_r from mixing with glueballs is corrected by the effect of the $\lambda\phi^4$ type interactions of Fig. 3a.

III. THE BREAKDOWN OF COLOR SYMMETRY IN THE BAG MODEL

The bag model gives a simple dynamical picture of how the breakdown of exact QCD color symmetry may be realized. Curiously, we find a picture which is midway between the arguments of Georgi [12] and those of DeRújula, Giles, and Jaffe [13] on the viability of a confined local gauge symmetry undergoing spontaneous symmetry breakdown. We present this here, although it suggests further lines of study and several points which we have yet to clarify. It may represent a mechanism of more general interest.

In Fig. (1), we show the mass of the 0^{++} bag state containing two colored constituent scalars versus the mass of the constituents. Branch I is the normal case in which $\mu^2>0$ and we find, as expected, that the bound state mass asymptotically approaches $2|\mu|$ as $\mu^2\to\infty$.

Branch II shows the bag mass plotted against $|\mu|$ for $\mu^2<0$ as is of interest in models in which QCD is spontaneously broken. We see that for $|\mu| \leq B^{1/4}$, the 0^{++} state is a hadronic bound state even though its constituents are tachyonic. Of course, this is due to the fact that the constituents kinetic energy is given by $\sqrt{(\pi^2/R_0^2)-\mu^2}$ and this remains real as long as $R_0<|\pi/\mu|$.

The 0^{++} bound state mass is determined by minimizing the energy,

$$E = \frac{4}{3}\pi R_0^3 B + 2\sqrt{\left(\frac{\pi^2}{R_0^2}\right)-\mu^2} \tag{3.1}$$

with respect to R_0, where we assume $Z_0=0$ for convenience. For sufficiently large R_0, this expression always becomes unphysical and we must distinguish between three cases. Let R_{min} be the value of R_0 which minimizes Re(E); then we have (a) $R_{min}<|\pi/\mu|$, (b) $R_{min}=|\pi/\mu|$, and (c) $R_{min}>|\pi/\mu|$. These cases are displayed schematically in Fig. (4). Clearly case (b) defines the critical value of $|\mu|$ for which no stable bound state exists for larger values of μ. This corresponds to the value $|\mu|_{crit}=2.7B^{1/4}$ on Branch II of Fig. (1). Thus for $|\mu|>|\mu|_{crit}=2.7B^{1/4}$ there is a clear indication of an instability in the bag model. This corresponds roughly to the expectations of Georgi who argues that such an instability is a non-calculable first order phase transition and occurs at too high a mass scale to correspond to a model of broken QCD. (The $|\mu|<|\mu|_{crit}$ states, however, are perfectly acceptable hadronic states.) Indeed, $|\mu_{crit}|=0.39$ GeV, which is too large a scale for the breakdown of $SU(3)_{color}$ into any residual subgroup.

However, the above analysis is incomplete. We have only studied the case of a bag containing tachyonic constituents with zero vacuum expectation values. We must also consider bag states in which the colored scalars have non-zero vacuum expectation values inside of the bag cavity. This leads to some novel consequences.

Consider a modified bag Hamiltonian for real scalar fields with an unstable Higgs potential:

$$H = \int_{\text{Bag Volume}} d^3\vec{r}\left\{ \left(\frac{\partial\phi}{\partial t}\right)^2 + \vec{\nabla}\phi\cdot\vec{\nabla}\phi - \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + B\right\} \quad , \qquad (3.2)$$

where we keep only the tree approximation terms in a general Higgs potential. Suppose we now let the ϕ field develop a vacuum expectation value, v, which minimizes the potential. Rewriting the Hamiltonian in terms of the shifted field $\hat{\phi}=\phi-v$ we find,

$$H = \int d^3\vec{r}\left\{\left(\frac{\partial\hat{\phi}}{\partial t}\right)^2 + \vec{\nabla}\hat{\phi}\cdot\vec{\nabla}\hat{\phi} + \frac{\mu^2}{2}\hat{\phi}^2 + B - \left(\frac{\mu^4}{2\lambda}\right) + \ldots\right\} \quad , \qquad (3.3)$$

where we now neglect the left-over $\hat{\phi}^3$ and $\hat{\phi}^4$ interactions. We see that the Hamiltonian of Eq. (3.3) describes a normal $\mu^2>0$ scalar field in a bag with a different bag constant $B'=(B-\mu^4/2\lambda)$. The resulting bag energy of a Φ bound state containing two scalar fields is then,

$$E = \frac{4}{3}\pi R_0^3\left(B - \frac{\mu^4}{2\lambda}\right) + 2\left(\frac{\pi^2}{R_0^2} + \mu^2\right)^{1/2} \quad . \qquad (3.4)$$

We must now distinguish between three new cases: (a) $B-(\mu^4/2\lambda)>0$, (b) $B-(\mu^2/2\lambda)=0$, and (c) $B-(\mu^4/2\lambda)<0$. The energy for these three cases is plotted in Fig. (5) for a fixed μ^2. In Fig. (5), we have assumed a value of $|\mu|<|\mu|_{crit}$. We now consider the resulting physics of the bag.

In case (c), we have $\mu>(2\lambda B)^{1/4}$. This can in principle occur for $|\mu|<<|\mu|_{crit}$ in which case there is no stable bag state containing scalars. The bag will expand to an infinite radius, (after perhaps tunneling through a local barrier at $R_{barrier}$), and the vacuum expectation value of the scalar field will be nonzero and constant everywhere. Furthermore, the two scalar fields now become liberated and the energy of the vacuum is clearly unbounded below. It is doubtful that there exist confined hadrons in this phase of the theory because inside of a candidate hadronic state ϕ will always have a vacuum expectation value and the effective bag constant will be negative. Indeed, we interpret case (c) as a completely broken non-confining theory with the vacuum totally rearranged.

Case (b), $\mu=(2\lambda B)^{1/4}$, is a deconfining phase containing both quasi-stable hadrons with energies situated at the minimum $R_{min}^{(1)}$ and with free infinite radius states in which the two scalar particles escape to infinity. (R_{min}^{1} is found by minimizing Eq. (3.4)).

Case (a), $\mu<(2\lambda B)^{1/4}$, is a novelty and we further distinguish two subcases, (ai) and (aii). In case (ai), we have a quasi-stable color singlet hadron with an energy minimum at $R_{min}^{(2)}$. At $R_{min}^{(2)}$, there appears to be a new bag with a bag constant $B-(\mu^4/2\lambda)$ and with an energy lower than the normal hadronic state. The scalar vacuum expectation value is localized to a scale $R_{min}^{(2)}$ and does not permeate all of space. The Higgs scalars seem to be confined within the larger bag of radius $R_{min}^{(2)}$. The details of the spectrum in this phase are of interest and we suspect that they are sensitive to the resulting masses of the vector bosons within the bag state. If the Compton wavelength of a massive gluon, λ_g, is small relative to $R_{min}^{(2)}$, the theory may appear to be a broken theory with a broken spectrum. Conversely, if $\lambda_g >> R_{min}^{(2)}$, the theory is evidently a peculiar kind of confining theory with no breakdown of color.

In case (aii), the energy minimum at $R_{min}^{(2)}$ is higher than that at $R_{min}^{(1)}$ and we suspect that the theory is unbroken QCD. Here the local energy minimum at $R_{min}^{(2)}$ must correspond to unstable states in which apparently massive gluons decay into lighter color singlet hadrons. Thus a "massive gluon" must here, in fact, be a color singlet bound state of gluons and scalars.

Our conclusion is that models of spontaneously broken QCD can exist for choices of $\mu<(2\lambda B)^{1/4}$ in which μ is comfortably less than $\mu_{crit} \simeq 2.7B^{1/4}$. These models are extremely sensitive to the Higgs quartic coupling as well as the bag coupling, B. Thus, the argument that μ_{crit} is determined solely by Λ_{QCD}, (i.e. by B as in Fig. (5)) is an oversimplification. Also, we find that there may exist systems which are novel in that they are midway between the totally confining and totally broken phases as in Fig. (5). In the next section, however, we put aside these speculative concerns and calculate the hadronic properties of scalar-scalar bound states.

IV. HADRONIC PROPERTIES

A. Hadronic Decay Widths

In this section, we estimate the hadronic widths of the $\Phi_r, 0^{++}$ bound states. In the bag model, such estimates are subject to large uncertainties due primarily to the absence of a high momentum scale which can control the QCD corrections. We will be forced to rely upon perturbation theory for momentum transfers of the order of 1 GeV. Nonetheless, as rough estimates these are probably reliable.

We expect that the decay of a Φ_r bound state is a typical OZI suppressed process as it involves the creation of two gluons and their subsequent decay to quark-antiquark pairs. The leading diagrams for a positive charge conjugation object are those of Fig. 6. For constituents in the r-dimensional color representation, we expect a width of order:

$$\Gamma(\Phi_r \to \text{hadrons}) \simeq \frac{1}{4}[T(r)/T(3)]^2[3/\dim(r)]\Gamma(\bar{q}q \to \text{hadrons})_{OZI}, \qquad (4.1)$$

where (1/4) is a spinology factor and the remaining factors correct the color normalizations. ($T(r)\delta_{ab} \equiv \sum_{ij}(\lambda^a_{ij}/2)(\lambda^b_{ji}/2)$ and $\dim(r)$ is the dimension of the representation). In Table 1, we present these naive scaling estimates of the widths for a 0^{++} state assuming a typical width $\Gamma(\bar{q}q \to \text{hadrons})_{OZI}$ of 1 MeV. As one would expect, the higher color representations such as the {21} and the {27} have quite large decay widths owing to the large color factors.

It is instructive to calculate the hadronic decay width of the Φ_r in the MIT bag model. The localization of the bag in position space introduces momentum uncertainities in the spatial wave function, $\phi(r)$, (we suppress the color indices for convenience here):

$$\phi(\vec{r}) = \int \frac{d^3\vec{p}}{2p_0(2\pi)^3} A(\vec{p}) e^{-i\vec{p}\cdot\vec{r}} \quad . \tag{4.2}$$

For the 0^{++} bound state, the Fourier transforms of the configuration space wavefunctions are readily performed:

$$A(\vec{p}) = NR_0^2 \frac{p_0}{|\vec{p}|} \frac{\sin(|\vec{p}|R_0)}{(\pi^2-|\vec{p}|^2R_0^2)} \quad , \tag{4.3}$$

where N is an irrelevant normalization factor and p_0 is the energy of the constituent; $p_0^2=\mu^2+\vec{p}^2$.

We define a two particle bag state centered at coordinate $\vec{X}_0$ to be,

$$|B,\vec{X}_0\rangle = \int \frac{d^3\vec{k}}{2k_0(2\pi)^3} \frac{d^3\vec{q}}{2q_0(2\pi)^3} A(k)A(q)a^+_{ki}a^+_{qj}|0\rangle \frac{\delta_{ij}}{\sqrt{\dim(r)}} e^{-i(\vec{k}+\vec{q})\cdot\vec{X}_0} \quad , \tag{4.4}$$

where (i,j) are color indices in the r-dimensional color representation and a^+_{kr} is the operator which creates $\phi_r(k)$ from the vacuum. The momentum eigenstates in the bag are superpositions of the states $|B,\vec{X}_0\rangle$,

$$|B,\vec{P}\rangle = \int d^3\vec{x}\; e^{i\vec{P}\cdot\vec{x}} \;\vec{F}(P)|B,\vec{x}\rangle \quad . \tag{4.5}$$

Here F(P) is given by,

$$F(\vec{P}) = \sqrt{2P_0}\,\{\int \frac{d^3\vec{K}}{2K_0(2\pi)^3}\,\frac{1}{2|P_0-K_0|}\,|A(\vec{K})|^2|A(\vec{P}-\vec{K})|^2\}^{-1/2} \quad , \tag{4.6}$$

which ensures the covariant normalization,

$$\langle B,\vec{P}|B,\vec{P}'\rangle = (2\pi)^3 2P_0\delta^{(3)}(\vec{P}-\vec{P}') \quad . \tag{4.7}$$

(Here P_0 is the energy of the bound state, $P_0=\sqrt{m_\Phi^2+\vec{P}^2}$).

In the rest frame, $\vec{P}=0$ and we have the amplitude for the hadronic decay of the 0^{++} bound state from the diagrams of Fig. 6:

$$A(\Phi_r\to gg) = \frac{-ig_s^2\mathrm{Tr}(\lambda^a/2\ \lambda^b/2)}{\sqrt{\dim(r)}}\int\frac{d^3\vec{p}}{2p_0(2\pi)^3}\,A(\vec{p})A(-\vec{p})F(0)B_{\mu\nu}(p,k,q)\epsilon_1^\mu\epsilon_2^\nu \ . \tag{4.8}$$

The kinematics are given by $P=(m_\Phi,0)=k+q$. (We assume that the hadronic decay width is equal to the width into two gluons.) Evaluating the Feynman diagrams of Fig. (6) gives:

$$B(p,k,q)_{\mu\nu}\epsilon_1^\mu\epsilon_2^\nu = 2\Big[\frac{p\cdot\epsilon_1 p\cdot\epsilon_2 - p\cdot\epsilon_1 k\cdot\epsilon_2}{-p\cdot k} + \frac{p\cdot\epsilon_1 p\cdot\epsilon_2 - p\cdot\epsilon_2 q\cdot\epsilon_1}{-p\cdot k} - \epsilon_1\cdot\epsilon_2\Big] \ , \tag{4.9}$$

where we have noted that $k^2=q^2=0$ and that the constituents are on-shell, $p^2=\mu^2$. Also we have $k\cdot\epsilon_1=q\cdot\epsilon_2=0$.

For the normalization factor F(0) we obtain,

$$F(0) = \sqrt{2m_\Phi}\,\{\int\frac{d^3\vec{K}}{4K_0^2(2\pi)^3}\ N^4\ \frac{K_0^4}{|\vec{K}|^2}\ \frac{\sin^4(|\vec{K}|R_0)}{(\pi^2-|\vec{p}|^2R_0^2)^4}\ R_0^8\}^{-1/2} = \frac{4\pi}{N^2R_0^3}\sqrt{\frac{m_\Phi}{R_0 I_1}} \quad , \tag{4.10}$$

where,

$$I_1 = \int_0^\infty \frac{\sin^4 x}{(\pi^2-x^2)^4}\, dx \simeq 1.6\times 10^{-3} \tag{4.11}$$

and we have assumed $\mu R_0 << 1$.

The momentum integrations of the amplitude in Eq. (4.8) are straightforward and we find for the decay amplitude into outgoing gluons with polarizations λ and λ',

$$A(\Phi_r \to gg) = \frac{-ig_s^2 \mathrm{Tr}(\lambda^a/2\ \lambda^b/2)}{2\pi^2} \sqrt{\frac{m_\Phi}{I_1 R_0}}\, (I_2 - \frac{I_3}{R_0|\vec{R}|})\, \varepsilon_1^{(\lambda)} \cdot \varepsilon_2^{(\lambda')}, \tag{4.12a}$$

where,

$$I_2 = \int_0^\infty \frac{\sin^2 x}{(\pi^2-x^2)^2}\, dx \simeq 7.96\times 10^{-2} \quad \text{, and} \tag{4.12b}$$

$$I_3 = \int_0^\infty \frac{x \sin^2 x}{(\pi^2-x^2)^2} \simeq .226 \quad . \tag{4.12c}$$

The resulting decay width is,

$$\Gamma(\Phi_r \to gg) = \frac{4\alpha_s^2}{\dim(r)}\, (T(r))^2\, \frac{1}{\pi R_0 I_1}\, (I_2 - \frac{2I_3}{m_\Phi R_0})^2 \quad . \tag{4.13}$$

These widths are listed in Table 1 for $\alpha_s = .5$, the value typically obtained in bag model fits to hadronic data. We note that the bag model decay widths are generally a factor of two larger than those obtained in our naive scaling model. We consider this to be an excellent agreement in view of the crudity of our scaling model.

B. Hadronic Production Cross Sections

The Φ_r bound states are expected to be produced in gluon fusion processes in such collisions as pp, $p\bar{p}$, and πp at very high energies. The production cross-section is given by:

$$\sigma_{\Phi_r} = \frac{\pi^2 \Gamma_r}{4m_\Phi^3} \int_\tau^1 dx \frac{f^1(x) f^2(\tau/x)}{(x+\tau/x)} \quad , \tag{4.14}$$

where $\tau = M_\Phi^2/s$, s is the total center of mass energy squared, x is the Feynman parameter, and $f^1(x)$, $(f^2(x'))$, is the gluon distribution of projectile 1 (2). Γ_r is the hadronic decay width calculated above for the appropriate bound state of scalars. We assume that the gluon distribution function in the proton is of the form [13]:

$$f^g_{proton}(x) = \frac{3}{x}(1-x)^5 \quad . \tag{4.15}$$

The cross sections for producing the Φ_r by gluon fusion in proton-proton interactions are shown in Table 2 for various representations r of the scalar constituents.

C. Spectrum of Bound States

In addition to the 0^{++} bound state, there will exist a full excitation spectrum whose properties are sensitive to the nature of the constituent's color representations and which involves a subtlety of the bag model, namely, the question of spurious states (those states which exist in the bag model, but not in the standard quark model). We discuss these questions for scalar-scalar bound states through J=2 and for the first radial excitation of the 0^{++} state.

Excited states are described by linear combinations of products of the general bag wavefunctions of eq.(2.3) for arbitrary ℓ, m, and k, where k labels the radial nodes. The spherical cavity approximation of the standard bag model treats the bag itself as a fixed object centered at a coordinate vector $\vec{X}_0$. Hence, the motion of two constituents, (located at $\vec{r}_1$ and $\vec{r}_2$ inside the bag), is an effective three body problem involving the relative coordinates $\vec{r}_1-\vec{X}_0$ and $\vec{r}_2-\vec{X}_0$. The constituents are assigned to orbitals relative to $\vec{X}_0$ and one obtains a much larger set of configurations than would emerge in a system described by a two-body potential which involves only the relative coordinate $\vec{r}_1-\vec{r}_2$. We refer to these additional configurations as "spurious states."

The problem has been discussed by Rebbi [14] and by DeGrand and Jaffe [15] for the standard bag model containing quarks and gluons. The latter authors suggest that the actual spectrum of hadrons should contain some vestige of the spurious states in the sense that the bag itself may be viewed as a physical component of a hadron. One might argue that a typical hadron has a substantial gluonic component, as is suggested by the deep inelastic energy-momentum sum rule, and that the bag represents a collective description of this gluonic component of the hadron, which can thus have its own inertia relative to the constituent quarks. This is an open question phenomenologically.

However, one should be careful to remove the spurious states in situations where they are clearly unphysical. Part of our focus is upon scalar constituents in large color representations. One might argue that for low dimensional color representations, it is as likely that a constituent will exchange a gluon with another constituent as with the collective gluonic medium of the bag. However, as the color representation becomes large, it is increasingly probable that constituent - constituent gluon exchanges will occur more frequently than constituent-bag gluon exchanges. The spurious states would fade into the background in this limit, if they exist at all. Furthermore, the properties of the bag states simplify considerably upon removing the spurious states because angular momentum is not a good quantum number for these states.

The constraints we impose to remove these spurious states are simple. Let $\vec{R} = (\vec{r}_1+\vec{r}_2)/2$ be the center of mass coordinate of the two scalar constituents and assume that the 0^{++} state of eq.(2.4) is the physical ground state of the bag. Then we may consider all multipole operators constructed from $\vec{R}$; for example, the dipole and quadrupole operators are:

$$d_i = R_i$$
$$Q_{ij} = R_i R_j - \frac{1}{3}\,\delta_{ij} R^2 \quad .$$

The physical states are those states which have no transitions induced among themselves by the R-multipole operators. Physically, any state having an R-multipole transition to another state is just a state describing the motion of a non-internally excited bound state relative to the bag coordinate $\vec{X}_0$.

We now consider the J=1 angular excitations of two scalar constituents whose normalized wavefunctions are the following:

$$\psi_m^{1(\pm)}(\vec{r}_1,\vec{r}_2) = N_1^{(\pm)}\{j_0(K_0^0 r_1)j_1(K_1^0 r_2)Y_{1m}(\Theta_2,\phi_2)$$

$$\pm j_0(K_0^0 r_2)j_1(K_0^0 r_1)Y_{1m}(\Theta_1,\phi_1)\} \quad . \tag{4.16}$$

$(K_i^P R_0)$ is the $(P+1)^{st}$ root of the Bessel function j_i. These two particle wavefunctions satisfy the Klein-Gordon equation in the bag and the boundary condition that the wavefunction vanish on the bag surface,

$$\psi_m^{1(\pm)}(\vec{R}_0,\vec{r}_2) = \psi_m^{1(\pm)}(\vec{r}_1,\vec{R}_0) = 0 \quad . \tag{4.17}$$

We may normalize the probability charge to one, which is equivalent to satisfying the energy-momentum conservation bag boundary condition of eq.(2.2c).

The wavefunction $\psi_m^{1(+)}$, $(\psi_m^{1(-)})$, corresponds to positive, (negative), parity and so for complex scalar constituents would also imply positive, (negative), charge conjugation. For a real color representation, the $\psi_m^{1(-)}$ state could not exist as a color singlet by Bose statistics. However, if we consider the matrix elements of $\psi_m^{1(\pm)}$ to the 0^{++} ground state via the dipole operator, we find:

$$\langle 0^{++}|\tfrac{1}{2}(\vec{r}_1+\vec{r}_2)|\psi_m^{1(\pm)}\rangle = N_1^{(\pm)} N\int d^3\vec{r}_1 d^3\vec{r}_2 [j_0(K_0^0 r_1) j_0(K_0^0 r_2)]$$

$$\cdot\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)[j_0(K_0^0 r_1) j_1(K_1^0 r_2) Y_{1m}(\Theta_2,\phi_2) \pm$$

$$j_0(K_0^0 r_2) j_1(K_1^0 r_1) Y_{1m}(\Theta_1,\phi_1)] \quad (4.18a)$$

$$= \begin{cases} 0 \quad \text{for} \quad \psi_m^{1(-)} & (4.18b) \\ \\ (\text{nonzero}) \quad \text{for} \quad \psi_m^{1(+)} & (4.18c) \end{cases}$$

These results follow simply from the symmetry of the integrals under the interchange of $\vec{r}_1$ and $\vec{r}_2$. Hence the 1^+ state, $(\psi_m^{1(+)})$, is a spurious state which corresponds to an $\ell=1$ excitation of the ground state relative to the bag coordinate $\vec{X}_0$ and not to an internal excitation. As one expects in a two-body potential, the J=1 vector state containing two neutral, real colored scalars in the color singlet state does not exist. The physical state is the odd charge conjugation state $\psi_m^{1(-)}$ consisting of two complex scalars. (Correspondingly, the internal dipole transition to the ground state through the operator $((\vec{r}_1-\vec{r}_2)/2)$ occurs only for $\psi_m^{1(-)}$, as it should for a bona fide excitation, while vanishing for $\psi_m^{1(+)}$.)

We now extend the preceeding arguments to higher excitations including the J=2, 1, and 0 orbital excitations. The following wavefunctions span these excitations and satisfy the Klein-Gordon equation in the bag and the bag boundary conditions:

$$\psi_m^{2(\pm)}(\vec{r}_1,\vec{r}_2) = N_2^{(\pm)}\{j_0(K_0^0 r_1)j_2(K_2^0 r_2)Y_{2m}(\Theta_2,\phi_2)$$

$$\pm j_0(K_0^0 r_2)j_2(K_2^0 r_1)Y_{2m}(\Theta_1,\phi_1)\} \tag{4.19a}$$

$$\psi_m^{2}(\vec{r}_1,\vec{r}_2) = N_2\{j_1(K_1^0 r_1)j_1(K_1^0 r_2)\sum_{m',m''} C^{2,m}_{1m',1m''} Y_{1m'}(\Theta_1,\phi_1)$$

$$Y_{1m''}(\Theta_2,\phi_2)\} \tag{4.19b}$$

$$\psi_m^{1}(\vec{r}_1,\vec{r}_2) = N_1\{j_1(K_1^0 r_1)j_1(K_1^0 r_2)\sum_{m',m''} C^{1,m}_{1m',1m''} Y_{1m'}(\Theta_1,\phi_1)$$

$$Y_{1m''}(\Theta_2,\phi_2)\} \tag{4.19c}$$

$$\psi^{0}(\vec{r}_1,\vec{r}_2) = N_0\{j_1(K_1^0 r_1)j_1(K_1^0 r_2)\sum_{m',m''} C^{0}_{1m',1m''} Y_{1m'}(\Theta_1,\phi_1)$$

$$Y_{1m''}(\Theta_2,\phi_2)\} \quad , \tag{4.19d}$$

where in the standard notation, the Clebsch-Gordan coefficients are:

$$C^{J,m}_{\ell_1 m_1,\ell_2 m_2} = \langle \ell_1 \ell_2 Jm | \ell_1 m_1 \ell_2 m_2 \rangle \quad . \tag{4.20}$$

Those states having even internal parity are $\psi_m^{2(+)}$, ψ_m^2, and $\psi^{\,0}$, while $\psi_m^{2(-)}$ and ψ^1 are odd parity states and cannot exist with constituent scalars in real color representations. To discover which are the spurious states we consider the center of mass dipole transitions between $\psi_m^{1(-)}$ and each of the states listed in eq.(4.19). As before, one finds trivially from the symmetries under the exchanges of $\vec{r}_1$ and $\vec{r}_2$ that this matrix element vanishes for ψ^0, ψ_m^2, and $\psi_m^{2(+)}$, while it is nonzero for ψ_m^1 and and $\psi_m^{2(-)}$. Hence, the odd internal parity objects are spurious and ψ^0, ψ_m^2, and $\psi_m^{2(+)}$ are potentially physical states.

However, it is insufficient to consider only the dipole transitions -- we must now consider the quadrupole matrix elements between ψ^0, ψ_m^2, and $\psi_m^{2(+)}$ and the 0^{++} ground state. We consider an arbitrary linear combination of $\psi_m^{2(+)}$ and ψ_m^2:

$$|\hat{\psi}_m^2\rangle \equiv \frac{\alpha|\psi_m^{2(+)}\rangle+\beta|\psi_m^2\rangle}{\sqrt{\alpha^2+\beta^2}} \quad , \tag{4.21}$$

and evaluate the matrix element of the quadrupole operator,

$$\langle 0^{++}|Q_{ij}|\hat{\psi}_m^{(2)}\rangle \equiv M_{ij} \quad . \tag{4.22}$$

By expanding the quadrupole operator in terms of spherical harmonics, we find the result:

$$M_{ij} \propto (\text{constant})\{\alpha 2\pi I_0 J_1 N_2^{(+)} + \beta J_2 N_2 \sqrt{\tfrac{5\pi}{6}}\} \quad . \tag{4.23}$$

where J_1, J_2, and I_0 are overlap integrals of Bessel functions:

$$J_1 = \int_0^{R_0} r^4 j_0(K_0^0 r) j_2(K_2^0 r) dr \quad , \tag{4.24a}$$

$$J_2 = \int_0^{R_0} r^3 j_0(K_0^0 r) j_1(K_1^0 r)dr \quad , \quad \text{and} \tag{4.24b}$$

$$I_0 = \int_0^{R_0} r^2 [j_0(K_0^0 r)]^2 dr \quad . \tag{4.24c}$$

Hence, the physical J=2 state for which the quadrupole transition vanishes is a normalized linear combination of $\psi_m^{2(+)}$ and ψ_m^2, where α and β are related by:

$$\alpha = -\sqrt{\frac{5\pi}{6}}\, \beta\left(\frac{J_2^2}{J_1 I_0}\right)\left(\frac{N_2}{2\pi N_2^{(+)}}\right) \quad . \tag{4.25}$$

The non-spurious wavefunction is then,

$$|\hat{\psi}_m^2\rangle = .44|\psi_m^{2(+)}\rangle - .88|\psi_m^2\rangle \quad . \tag{4.26}$$

We also evaluate the quadrupole moment between the ground state and ψ_m^0 to find,

$$\langle 0^{++}|Q_{ij}|\psi_m^0\rangle = 0 \quad . \tag{4.27}$$

Thus the ψ^0 state has no center of mass quadrupole transition to the ground state and so is a physically acceptable state. (This result is obvious by rotational invariance of the spin zero states).

Radial excitations can be treated in a similar fashion to the angular excitations considered above. The first radial excitation of the 0^{++} ground state corresponds to the wavefunctions,

$$\psi_R^{(\pm)}(\vec{r}_1,\vec{r}_2) = N_R^{0(\pm)}\{j_0(K_0^0 r_1)j_0(K_0^1 r_2) \pm j_0(K_0^1 r_1)j_0(K_0^0 r_2)\} \quad . \tag{4.28}$$

Again, upon considering the dipole transition between $\psi_R^{(+)}$ and $\psi_m^{1\ (-)}$ one readily finds that $\psi_R^{(-)}$ has a non-zero transition matrix element, while $\psi_R^{(+)}$ does not. Hence $\psi_R^{(+)}$ is the physical radial excitation while $\psi_R^{(-)}$ is spurious.

It is straightforward to extend this analysis to the 1^{--} radial excitations and beyond. In terms of two-body orbitals these physical states have simple interpretations. In Table (3) we summarize the above analyses and include estimates of the masses.

D. Pseudoquarks

In the context of the bag model, there is nothing to prevent an ordinary quark from forming a color singlet bound state with one of the scalars, ϕ_r. It should be emphasized that these are short distance bound states which should be produced readily in hadronic interactions. The bound states formed with a scalar octet, $(\phi_8 q\bar{q})$, and with a 27-plet, $(\phi_{27} qq\bar{q}\bar{q})$, are electrically neutral and could escape detection if they are short-lived. (The best experimental bound [16] on neutral particles with masses between 1 and 15 GeV is from an experiment which was only sensitive to particles with lifetimes greater than 10^{-7} seconds.) However, the bound states formed with the {3}, {6}, {10}, {15}, and {21} dimensional scalar particles all have fractional charges and masses between one and two GeV and should have been seen in short-distance processes -- they would be produced copiously in e^+e^- interactions if they existed.

V. SOME PHENOMENOLOGY AND CONCLUSIONS

The phenomenology of bound states of two scalar particles will be similar to that of the 0^{++} glueballs. Indeed, if a candidate Φ_r state is discovered, it will be extremely difficult to distinguish it from the 0^{++} glueball predicted by QCD.

There are only a handful of 0^{++} states which have been observed experimentally and they can be more or less understood in terms of the quark model. The isoscalar $\delta(980)$ and the isovector $S^+(980)$ have been described as $qq\bar{q}\bar{q}$ "crypto-exotic" states and the bag model predictions fit the spectrum well.

An obvious place to search for the $\Phi_r, 0^{++}$ states is in the radiative decays of the ψ and the Υ. The amplitudes for the radiative decay of a ψ to a 0^+ state and to a 0^- state are identical to lowest order in perturbation theory and so the rate of the decay of the ψ(or Υ) to $\Phi_r\gamma$ can be estimated by scaling the decay width from that of $\psi\to\eta\gamma$ or $\psi\to\eta'\gamma$. The η and η' are each an admixture of flavor SU(3) singlet and octet components. We assume exact SU(3) flavor symmetry and scale $\Gamma(\psi\to\Phi_r\gamma)$ from $\Gamma(\psi\to\eta'\gamma)$:

$$\Gamma(\psi\to\Phi_r\gamma) \simeq \frac{1}{4}(T(r)/T(3))^2 \frac{(1-m_\Phi^2/m_\psi^2)}{(1-m_{\eta'}^2/m_\psi^2)}\Gamma(\psi\to\eta'\gamma) \quad . \tag{5.1}$$

Using the experimental result $\Gamma(\psi\to\eta'\gamma)$ = 157 eV, we obtain for example, $\Gamma(\psi\to\Phi_{27}\gamma)$ = 104 KeV and $\Gamma(\psi\to\Phi_3\gamma)$ = 36.1 eV. With the exception of $\Gamma(\psi\to\Phi_3\gamma)$, the widths of the ψ into $\Phi_r\gamma$ are all much wider than those observed experimentally in $\Psi\to\eta\gamma$, $\psi\to\eta'\gamma$, etc. If our mass estimates are reliable and the mass of a Φ_r state is less than three GeV, then these states should be seen as resonances in the inclusive photon spectrum of the Crystal Ball.[8]

The bin width of the inclusive photon spectrum of the Crystal Ball experiment is proportional to the resolution, δE, of the sodium iodide shower detectors, where

$$\delta E = .028 E^{3/4} \tag{5.2}$$

(E is the photon energy measured in GeV.) At a photon energy of 1 GeV, the energy resolution, δE, is then 28 MeV. The predicted widths of $\psi\to\Phi_r\gamma$ are all much less than δE, so $\psi\to\Phi_r\gamma$ would be visible as a sharp spike in the inclusive photon spectrum. These decays are not seen at the predicted rates.

The Crystal Ball collaboration has looked at $\pi\pi$, KK, $\eta\eta$, etc. final states and again sees no evience at present for the existence of 0^{++} states. (One might worry that somehow the Φ_r is "hidden" under the f peak in $\psi\to\gamma f(1270)$. However, the branching ratio for $\psi\to\gamma f$ is much less, [B.R.$(\psi\to\gamma f) = 1.3\times10^{-3}$], than that predicted for $\psi\to\gamma\Phi_r$, (with the exception of r=3), and the angular distribution of photons is well fit by that expected for a spin 2 object. Indeed, the Crystal Ball

Collaboration claims that the relative probability for the spin 0 hypothesis as compared to the spin 2 hypothesis is 10^{-11}). The Crystal Ball data provides no evidence for the existence of bound states of scalars if the scalars have a dimension greater than three and if the mass of the bound state is less than 3 GeV. Further exploration of this mass range is of interest.

The radiative decays of the Υ should also prove a fertile ground for searching for the Φ_r if they are too massive to be produced in ψ decays. If any of the Φ_r states exist with masses less than 9 GeV, they should be seen at CESR.

In conclusion, we have examined the phenomenology of bound states of light, colored, electrically neutral scalars in varying representations r of SU(3). The existance of such scalar particles is highly constrained by current experimental limits. Color triplets are forbidden because they would form fractionally charged bound states with one antiquark--the naive bag model predicts that these states would be at 1.4 GeV and experimentally no such states are seen in e^+e^- interactions up to masses of 15 GeV. Colored sextets of scalars are also not allowed because of the non-existance of their bound states with an antiquark pair. Scalar particles in the {6}, {8}, {10}, {15}, {21}, and {27} dimensional representations of SU(3) should have been seen by the Crystal Ball unless the scalar-scalar bound states have masses greater than 3 GeV. Finally, scalar particles in higher dimensional representations than the {27} are forbidden by the requirement that QCD with three families of fermions be asymptotically free.

An important question to resolve is how reliable these bound state mass estimates are. The naive bag model value for the mass of a two scalar bound state is about 1.4 GeV. All of the corrections to the mass which we have included, (one gluon exchange and the zero-point energy), lower the energy of the bound state. It is possible unfortunately that the annihilation graph and quartic interaction of Fig. 2 make our estimates of the bound state masses unreliable. However, we suspect that even including such corrections rigorously, the mass scale associated with a bound state of two massless scalar fields must be a typical hadronic scale. In this case, the Φ_r, if they exist, should be seen in the ψ and/or Υ radiative decays.

Our conclusion is that if QCD is broken by colored scalars, then the short distance bound states of scalars should be readily produced in accelerators and the phenomenology of these scalars is already severely restricted by experimental limits.

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TABLE CAPTIONS

Table 1: Hadronic decay widths for a bound state, Φ_r, of two scalars in the r-dimensional representation of SU(3) are calculated in both a naive scaling model and in the MIT bag model.

Table 2. Production cross sections for producing Φ_r in pp collisions via gluon fusion are shown for varying center of mass energies. We assume a gluon distribution function in the proton of the form $f_g^p(x)=3(1-x)^5/x$.

Table 3: The scalar-scalar bound state spectrum in the MIT bag model is presented along with the two body correspondence of each state.

TABLE 1. Hadronic Decay Widths

r	$\Gamma(\Phi_r \to \text{hadrons})$ (MeV)	
	Naive Scaling	Bag Model
3	0.25	0.63
6	3.1	7.8
8	3.4	8.4
10	17	42
15	20	50
21	175	436
27	81	200

TABLE 2. PP Production Cross Sections

(σ in μb)			
r	$\sigma(\sqrt{s}=4$ GeV)	$\sigma(\sqrt{s}=11$ GeV)	$\sigma(\sqrt{s}=45$ GeV)
3	0.2	1.19	5.38
6	2.48	14.73	66.61
8	2.67	15.87	71.73
10	13.33	79.33	358.67
15	15.87	94.44	426.98
21	138.41	823.56	3723.30
27	63.49	377.78	1707.94

TABLE 3. Bound State Spectrum

State	Two-Body Correspondence	Mass (GeV) ($Z_0=0$)	Mass (GeV) ($Z_0=-2$)
$\psi_{0^{++}}$	1S (0^{++})	1.44	1.08
$\psi^{1(-)}$	2P (1^{--})	1.67	1.33
$\hat{\psi}^{2}$	3D (2^{++})	1.88	1.55
$\psi^{(0)}$	3S (0^{++})	1.88	1.55
$\psi_R^{(+)}$	2S (0^{++})	1.95	1.63
$\hat{\psi}_R^{1(-)}$	3P (1^{--})	2.17	1.86

FIGURE CAPTIONS

Fig. 1: The energy E of the 0^{++} bag state containing two colored scalars plotted as a function of the constituent mass μ for $\mu^2<0$ and for $\mu^2>0$. (E_{crit} is the energy at $\mu_{crit}=2.7\ B^{1/4}$).

Fig. 2: One gluon exchange graphs which contribute to the mass of the Φ_r bound state.

Fig. 3: (a) Contribution of the $\lambda\phi^4$ interaction to the Φ_r mass.

(b) Lowest order contributions to the mixing between Φ_r and the 0^{++} glueball.

Fig. 4: The energy, E(R) of a bag of radius R containing two scalars. Branch (a) corresponds to $R_{min}<|\pi|\mu|$, branch (b) to $R_{min}=|\pi|\mu|$, and branch (c) to $R_{min}>|\pi|\mu|$. (The constituent mass μ is fixed.)

Fig. 5: The energy, E(R) of a bag of radius R containing two scalars. Branch (a) corresponds to $B-(\mu^2/2\lambda)>0$, branch (b) to $B-(\mu^2/2\lambda)=0$, and branch (c) to $B-(\mu^2/2\lambda)<0$. Branch (ai) and (aii) both have hadrons bound in a bag of radius $R_{min}^{(2)}$. Branch (bi) corresponds to hadrons bound in a bag of radius $R_{min}^{(1)}$, while branch (bii) describes hadrons which are not bound in a bag.

Fig. 6: Lowest order contributions to the decay $\Phi_r\to 2$ gluons.

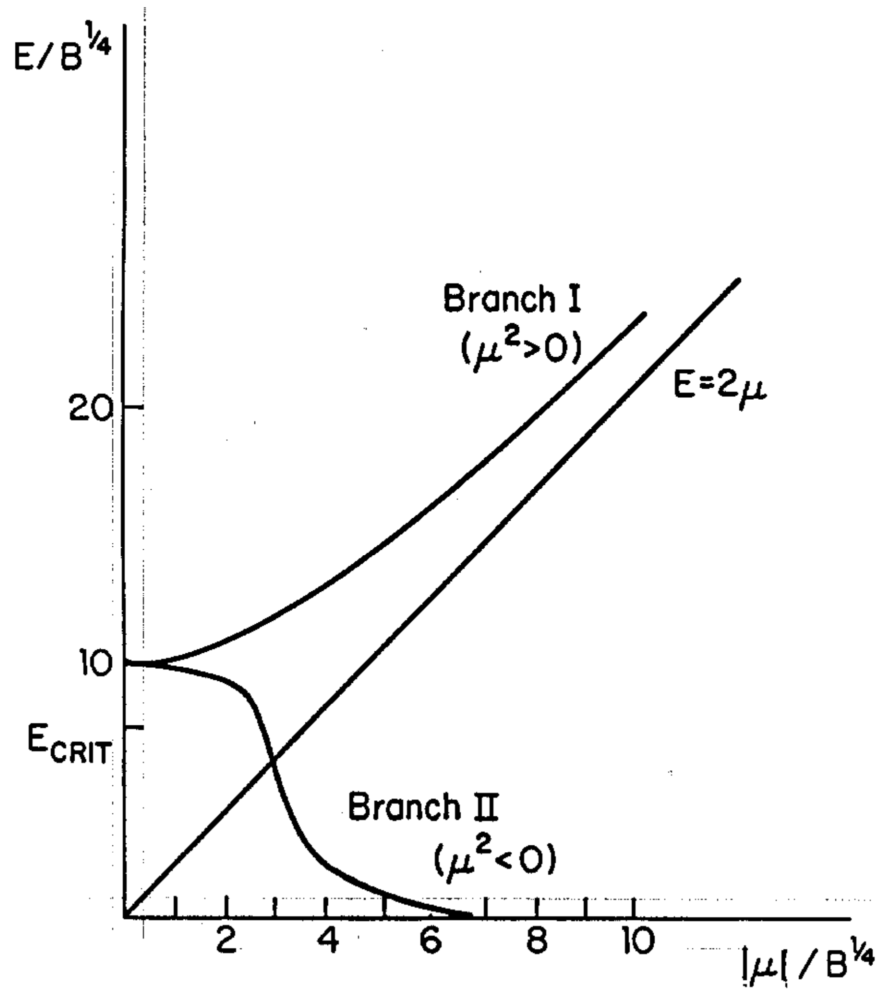


Fig. 1

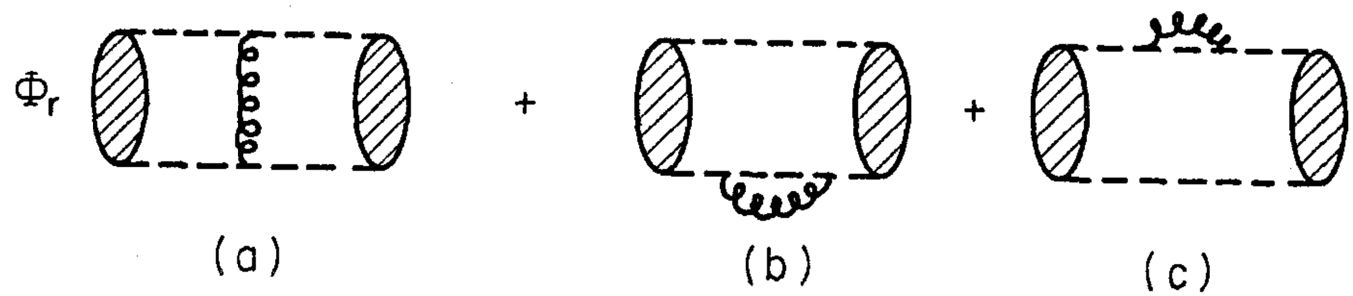


Fig. 2

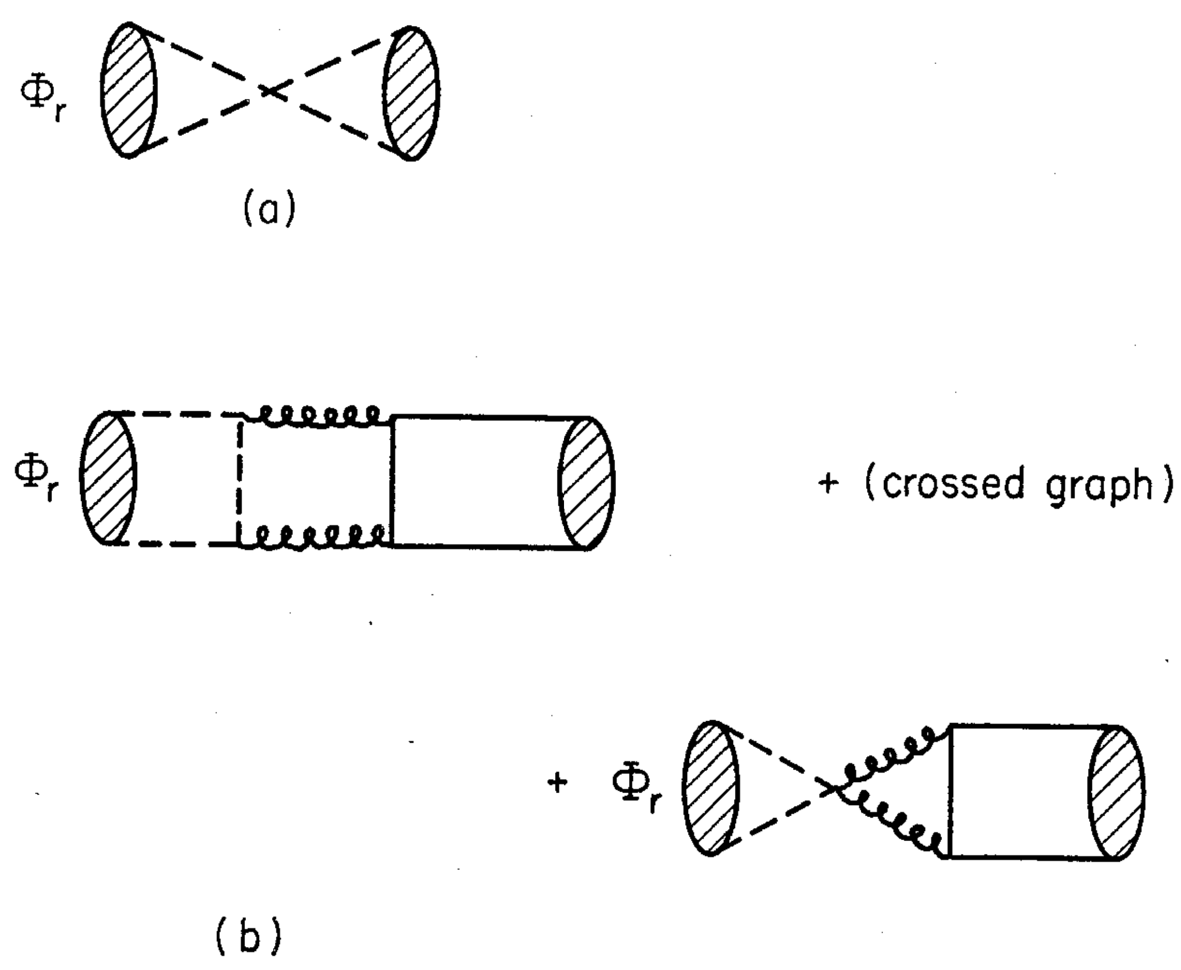
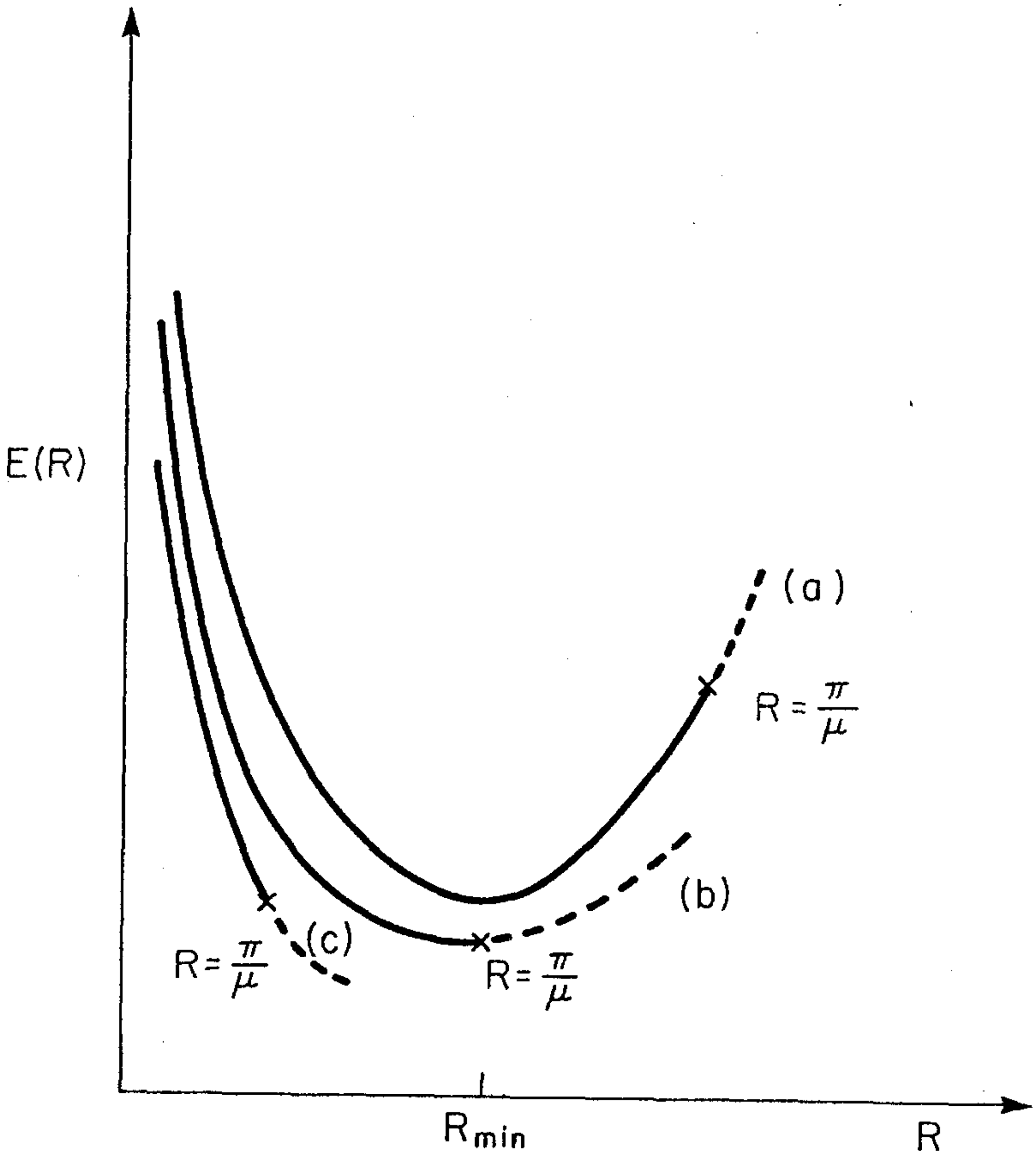


Fig. 3



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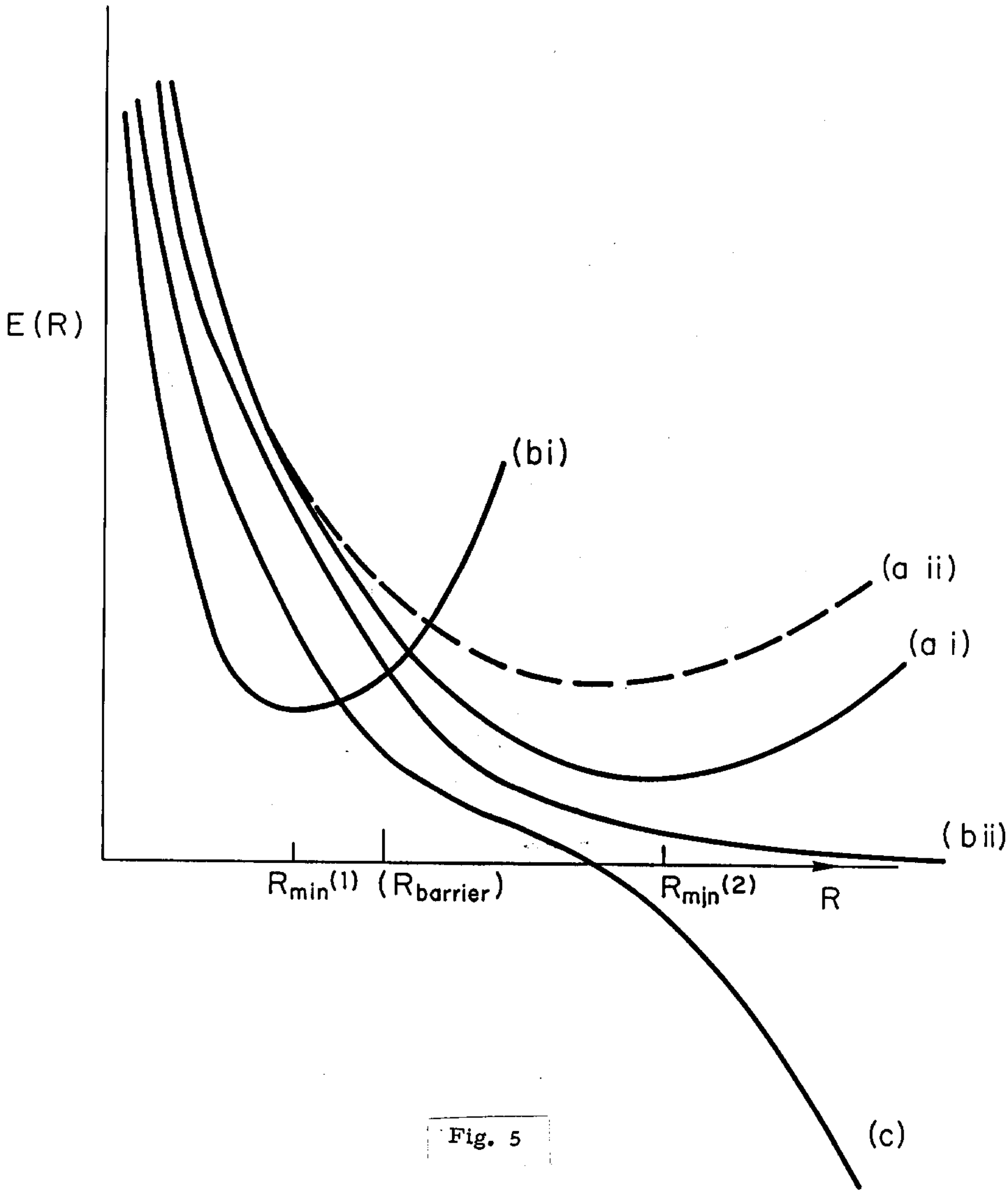

Fig. 5

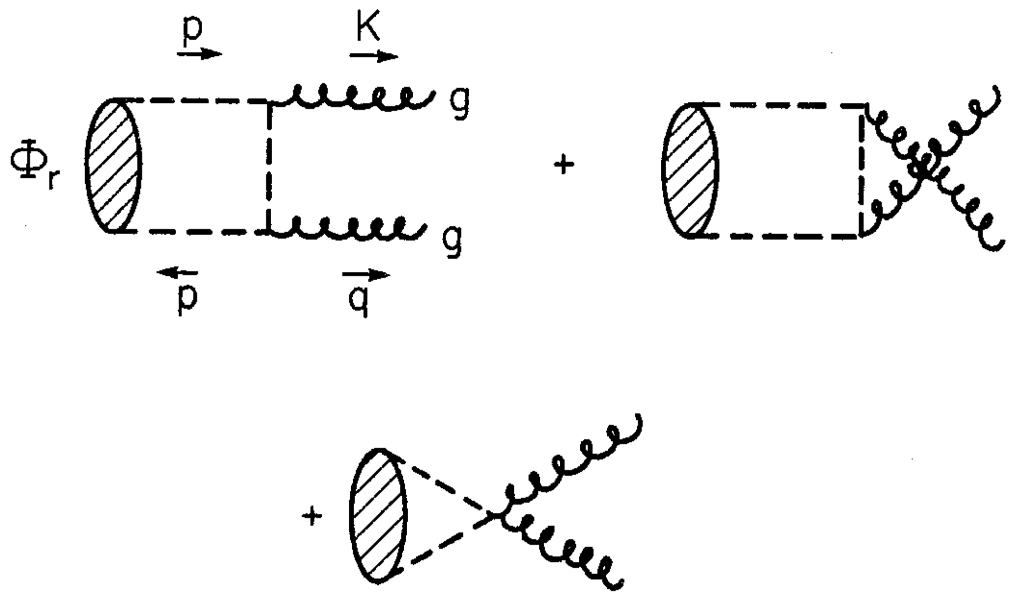

Fig. 6